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Two Syllogisms in the *Mozi*: Chinese logic and language

Byeong-uk Yi

University of Toronto

Abstract: This paper examines two syllogistic arguments contrasted in an ancient Chinese book, the *Mozi*, which expounds doctrines of the Mohist school of philosophers. While the arguments seem to have the same form, one of them (the one-horse argument) is valid but the other (the two-horse argument) is not. To explain this difference, the paper uses English plural constructions to formulate the arguments. Then it shows that the one-horse argument is valid because it is an instance of the plural cousin of a standard form of valid categorical syllogisms (*Plural Barbara*), and argues that the two-horse argument involves equivocal uses of a key predicate (the Chinese counterpart of 'have four feet') that has the distributive/non-distributive ambiguity. In doing so, the paper discusses linguistic differences between Chinese and English and explains why the logic of plural constructions is applicable to Chinese arguments that involve no plural constructions.

Keywords: Chinese logic, the *Mozi*, categorical, syllogism, Barbara, Plural Barbara, singular/plural, distributive/collective, equivocation, plural logic, mass noun thesis, semantics, Chinese count noun

The *Mozi* is an ancient Chinese book that expounds doctrines of the Mohist school of philosophers, a school founded by Mozi (ca. 479–381 BCE) that flourished in competition with Confucianism during the Warring States period (475–221 BCE). Mozi is well-known for rejecting the Confucian emphasis on family relations in morality by holding the doctrine of universal love (*jian ai*), love for all people without partiality based on family relations. In addition to chapters expounding Mohist moral and political doctrines, the *Mozi* has dialectical chapters that discuss science, logic, and philosophy of language among other subjects.[1] The last of those chapters, the *Smaller Selection* (*Xiao Qu*), presents various pairs of apparently parallel arguments one of which is correct (or valid) and the other incorrect (or invalid). I discuss in this paper one of those pairs, a pair of arguments contrasted in the last passage of the chapter. The passage contrasts two arguments by contrasting what holds for *two horses* with what holds for *one horse*. But both arguments seem to have a valid form of categorical syllogisms.[2] I aim in this paper to clarify the reason for the logical difference between the two arguments.

To analyze the arguments, one might begin by formulating them in English. This approach assumes that we can find suitable English counterparts of the Chinese arguments. But Chinese differs from English (and other familiar European languages) in lacking a *grammatical number*

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[1]They are Chapters 40–45 of the *Mozi*. See Mo (2010), which has the full text of the *Mozi* with Ian Johnston's translation; Graham (1978), which has the text of the dialectical chapters (*ibid*., 499–525) with his translation; and the online Mo (2006–2017). See also Mo (2013), which has Johnston's translation. For a general introduction to Mozi and Mohism, see Fung (1948, Chapter 5; 1952, Chapter 5) and Fraser (2015). For accounts of the Mohist logic and philosophy of language, see Graham (1978), Harbsmeier (1998, 326–337), Robins (2010), and Fraser (2017).

[2]The *Smaller Selection* has nine sections, and the passage contrasting the two arguments figures in the last paragraph of the last section. In addition to the translations listed in note 1, see Johnston (2000, 394–398) and Robins (2010, 248–256) for translations of the *Smaller Selection*.

system, which includes the *singular* and *plural* forms of nouns and predicates. For this reason, usual translations of the arguments obscure the parallelism between them. We can avoid this problem by using *plural constructions* (e.g., 'horses', 'have four feet') to obtain suitable formulations of the arguments in English. By doing so, we can see that one of the two arguments has a valid form, what I call *Plural Barbara*. This is the plural cousin of Barbara, a standard form of valid categorical syllogisms. And we can see that the other argument, which apparently has the same form, involves equivocal uses of a key predicate (the predicate amounting to 'have four feet') that has the *distributive*/*non-distributive* ambiguity. So we can explain the logical difference between the two arguments by applying a system of logic that can explain logical relations among plural constructions, such as *plural logic*.[3]

1. The Horse Passage: One horse and two horses

Call the passage in the Mozi that contrasts the two arguments in question the *Horse Passage*, for the arguments concern what holds for one horse and what holds for two horses. One would usually translate the passage roughly as follows:

> [H1] One horse is a horse; two horses are [also] horses. [And] A horse has four feet, which means that one horse has four feet, [but] not that two horses have four feet. .

[3]Plural logic, which extends elementary logic to explain logical relations among plural as well as singular constructions, explains the logic of plural constructions without reducing them to singular constructions. For this approach to the logic of plural constructions, see, e.g., Yi (1999; 2002; 2005; 2006; 2016), Rayo (2002), McKay (2006), Oliver and Smiley (2016), and Linnebo (2017).

. . one [of these] is correct and one is not.[4]

The remark "one is correct and one is not", on this translation, suggests a contrast between two groups of statements:

(1) a. One horse is a horse.

b. A horse has four feet.

c. One horse has four feet.

(2) a. Two horses are horses.

b. A horse has four feet. (=(1b))

c. Two horses have four feet.

(1a)–(1b) imply (1c). By contrast, (2a)–(2b) do not imply (2c). Although both (2a) and (2b) are true, (2c) is false—two horses have eight feet (with four each), not four.

Those who rely on the above translation of the Horse Passage, [H1], might hold that the logical difference between the arguments contrasted in the passage stems from the *singular*/*plural* distinction. On this account, the arguments have different logical forms. For the first argument (i.e.,

[4]All translations of Chinese (including the above translation of the Horse Passage and the translation given below, [H2]) are mine. See Graham (1978, 492 & 523–4), Mo (2010, 634), and Mo (2006–2017) for the Chinese text of the passage, and Graham (1978, 493), Johnston (2000, 387), Mo (2010, 635; 203, 296), Mo (2006–2017), and Robins (2010, 256) for other translations. The Horse Passage is followed by its sequel that contrasts two other arguments that also concern *one horse* and *two horses* (the sequel figures in the passage omitted in the above translation). The sequel is discussed in Appendix 3.

the argument consisting of (1a)–(1c)) involve only singular constructions (e.g., 'a horse', 'has'), but the second (i.e., the argument consisting of (2a)–(2c)) involves plural constructions (e.g., 'horses', 'have').

Note that (1a)–(1c) involve implicit universal quantification.[5] By making this explicit, we can paraphrase the statements as follows:

(1) a′. Anything that is one horse is a horse.

b′. Anything that is a horse has four feet.

c′. Anything that is one horse has four feet.

All of these have the universal affirmative form (in short, the **A** form) of categorical statements:

> **A** (the *standard **A** form*): Anything that is-*P* is-*Q*, where 'is-*P*' and 'is-*Q*' stand in for predicates of the singular form (e.g., 'is a horse', 'is white', 'runs fast').

And they form the premisses and conclusion of a valid categorical syllogism. The argument consisting of them has the form of Barbara:

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[5]The same holds for (2a)–(2c) except that (2a) and (2c) involve (implicit) *plural* universal quantification ('Any things are such that . . .') rather than the usual, *singular* universal quantification ('Anything is such that . . .'). See (5a)–(5c) below. Fung (2007, 527) takes *yi ma* ('one horse') to figure in the Chinese counterparts of (1a) and (1c) as a definite noun phrase amounting to '*that* one horse'. But the passage suggests no specific horse the definite noun phrase would refer to. It is more plausible to regard the statements as implicit universal quantifications amounting to (1a′) and (1c′) or the like (e.g., (4a) and (4c)) (see below for (4a) and (4c)).

Barbara:

Any thing that is-*P* is-*Q*.

Any thing that is-*Q* is-*R*.

:. Any thing that is-*P* is-*R*.

By contrast, (2a) and (2c) involve plural constructions and cannot be paraphrased by statements of the **A** form, for nothing (i.e., no one *thing*) is two horses while some *things* (e.g., Chitu and Dilu[6]) are two horses.[7]

Tan (1964, 408f) might suggest this account of the logical difference between the arguments contrasted in the Horse Passage. But I do not think it is a good account. The account invokes the singular/plural difference between (1a) and (1c), one the one hand, and (2a) and (2c), on the other. But this difference is an artifact of the English translation, [H1], for Chinese does not have a grammatical number system, as Graham (1978, 493) and Johnston (2000, 398) rightly note.

Consider, e.g., (1a) and (2a). We cannot get (2a) from (1a) by simply replacing 'one' with 'two', for the English statements have a syntactic difference: (1a) involves only singular constructions (e.g., 'is', 'a horse'), but (2a) involves plural constructions (e.g., 'are', 'horses'). But this difference does not arise from any difference between the Chinese originals:

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[6]Chitu and Dilu are famous horses of the Three Kingdoms period (AD 220–280) in Chinese history, the setting of the novel *Romance of the Three Kingdoms*.

[7]So 'Anything that is two horses has four feet', unlike (2c), is vacuously true. (Some might hold that (2c) can be paraphrased as 'Anything that consists of two horses has four feet', but applying the same scheme of paraphrase to (2a) yields a false statement: 'Anything that consists of two horses is a horse.')

(3) a. *Yi* *ma* *ma* *ye*.[8] [one horse horse ASN][9]

b. *Er* *ma* *ma* *ye*. [two horse horse ASN][10]

(1a) and (2a) figure in [H1] as translations of (3a) and (3b), respectively. Unlike the English translations, however, (3a) and (3b) draw complete syntactic parallels. And we can get (3b) from (3a) by simply replacing the numeral *yi* for one with the numeral *er* for two (and *vice versa*). The singular/plural difference between (1a) and (2a) arises because English has a grammatical number system, which includes the singular/plural morphology. English does not simply use nouns or verbs when they figure in statements, but rather their singular or plural forms: (1a) and (1b) have the singular and plural forms of the noun 'horse', respectively, as they have the singular and plural forms of the verb 'be' (i.e., 'is' and 'are').[11] Like a variety of languages (e.g., Japanese, Korean, Tagalog), however, Chinese has no singular or plural forms of nouns or verbs because it does not have a grammatical number system. The language simply uses nouns and verbs themselves where languages with such systems would use singular or plural forms of their counterparts. Thus the

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[8]In (3a)–(3b), the second *ma* ('horse') figures as a predicate amounting to 'is a horse' or 'are horses'. Chinese common nouns (e.g., *ma*), in the predicate position, can form predicates without addition of copulas (e.g., *nai*), although copulas might be added for special purposes (e.g., emphasis), as in *nai ma* ('is a horse, are horses' [be horse]). See, e.g., Dobson (1974, 68f). The sentence final particle *ye* usually figures as an assertion marker (in short, ASN).

[9]Square brackets are used for the English glosses of Chinese characters.

[10]Note that the noun *ma* ('horse') combines directly with numerals (*yi* for one and *er* for two) in (3a)–(3b). This conflicts with the thesis about Chinese nouns that Hansen (1983; 1992) holds: all Chinese common nouns are mass nouns (the *mass noun thesis*). See Appendix 2.

[11]The noun 'horse' is different from (albeit homonymous with) its singular form, and (1a), for example, has the singular form in both 'one horse' and 'a horse', not the noun itself.

Chinese noun *ma*, which amounts to the English noun 'horse' (not its singular or plural form), serves the roles of both of its forms, and figures in (3a) and (3b) without taking a singular or plural form.[12] Similarly, it figures without taking a singular or plural form in the Chinese counterparts of (1c) and (2c). Thus the two arguments contrasted in the Horse Passage draw syntactic parallels, and we can get the second argument from the first by replacing the numeral *yi* for one with a numeral for two.[13]

Does this mean that English, which has a grammatical number system, has no counterparts of the Chinese arguments that give rise to the same problem? I think not.

For the purpose of considering its logic, we can translate the Horse Passage as follows:

> [H2] Any things that are one horse are horses; any things that are two horses are [also] horses. [And] Any things that are horses have four feet, which means that any things that are one horse have four feet, [but] not that any things that are two horses have four feet. . . . one [of these] is correct and one is not.

Then the passage suggests a contrast between two arguments whose premisses and conclusions can be formulated in English as follows:

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[12]For the same reason, *zu* ('foot') does not take a plural form in the Chinese counterpart of (1b).

[13]The first premiss and conclusion of the second argument in the Horse Passage has different Chinese characters for two (*er* and *liang*). But the characters are interchangeable, and we can ignore the lack of parallelism between the two arguments that arises from the use of different characters for two. See Appendix 1.

(4) a. Any things that *are one horse* are horses.[14]

b. Any things that are horses have four feet.

c. Any things that *are one horse* have four feet.

(5) a. Any things that *are two horses* are horses.

b. Any things that are horses have four feet. (=(4b))

c. Any things that *are two horses* have four feet.

Call the first argument, the argument matching (4a)–(4c) (or their Chinese counterparts), the *one-horse argument*, and the second, the argument matching (5a)–(5c) (or their Chinese counterparts), the *two-horse argument*.

The two arguments, on the above formulation, have the same structure and we can get the two-horse argument from the one-horse argument by replacing '(are) one horse' in (4a) and (4c) with '(are) two horses'. While the one-horse argument is valid, however, the two horse-argument is not—(5a) and (5b) do not imply (5c) because any two horses have eight feet while any horses have four feet. What gives rise to this difference?

2. Plural Barbara

We can see that the one-horse argument is an instance of a valid form of categorical syllogisms. Its

[14]Some might take 'are one horse' (unlike 'is one horse') to be ill-formed, but the predicate phrase figures in, e.g., 'Dilu and Yuyan *are* not *two horses*, but one horse', which is true ('Yuyan' is another name of Dilu, as 'Tully' is another name of Cicero).

premisses and conclusion (i.e., (4a)–(4c)) are a kind of *plural* categorical statements. They are plural universal affirmative statements, namely, instances of the *plural cousin* of the standard **A** form of categorical statements (in short, the **A*** form):

> **A*** (the *plural* **A** *form*): Any things that are-*P* are-*Q*, where 'are-*P*' and 'are-*Q*' stand in for predicates of the plural form (e.g., 'are horses', 'are white', 'run fast').

So the argument is an instance of Plural Barbara, the plural cousin of the standard, singular form of Barbara:

> *Plural Barbara*:
>
> Any things that are-*P* are-*Q*.
>
> Any things that are-*Q* are-*R*.
>
> :. Any things that are-*P* are-*R*.

We can get the argument by replacing 'are-*P*', 'are-*Q*', and 'are-*R*' with 'are one horse', 'are horses', and 'have four feet', respectively.

Some instances of the plural **A** form (e.g., 'Any things that are horses are equine') are commonly used interchangeably with their singular cousins of the standard, singular **A** form (e.g., 'Anything that is a horse is equine'). But they do not exhaust all instances of the plural form. Its instances include plural constructions that are not equivalent to their singular cousins or do not have well-formed singular cousins: 'Any things that are two horses are horses' (=(5a)), 'Any things that

are two horses have four feet' (=(5c)), 'Any things that are brothers have the same surname',[15] 'Any things that are brothers with the same mother have different birth days', etc. So instances of Plural Barbara include arguments consisting of **A*** statements of the second kind, such as (6):

(6) Any things that are two brothers with the same parents are two siblings with the same mother.

Any things that are two siblings with the same mother have different birth days.

:. Any things that are two brothers with the same parents have different birth days.[16]

Although this argument does not fall under the standard singular form of Barbara, we can see that it is a valid argument. And we can see that Plural Barbara, like its singular cousin, is a valid form of categorical syllogisms.[17] This explains the validity of the one-horse argument.

If so, why is the two-horse argument not valid? Is it not a counterexample to the validity of Plural Barbara? I think not. The argument is not a proper instance of the form, for it involves equivocal uses of the predicate 'have four feet'. I explain this in the next section.

3. The distributive/non-distributive ambiguity

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[15]This is a variant of the main premiss of Argument I in Church (1956, 1): 'Brothers have the same surname.'

[16]The second premiss and conclusion of (6) are false, but this does not affect its validity.

[17]We can use the logic of plural quantification to show that Plural Barbara is a valid form, as we can use the logic of singular quantification to show that Barbara is a valid form. See, e.g., Axioms 6–8 in Yi (2006, 263).

Consider two horses: Chitu and Dilu. How many feet do they have? There are two possible answers:

(7) a. They have four feet.

b. They have eight feet.

Both are in a way correct, for the two horses *each* have four feet and have eight feet *in total*. This does not mean that they are both correct answers to the same question, but that they can be given as correct answers to different questions:

(8) a. How many feet do they *each* have?

b. How many feet do they have *in total*?

If so, the statement 'How many feet do they have?' is ambiguous. One might use it to raise either the first question or the second. And these questions have different answers: (7a) can be given as the correct answer to (8a), and (7b) to (8b). Moreover, (7a) and (7b) are also ambiguous. One might use (7a) to answer not only (8a) but also (8b), for it is ambiguous between (7a′) and (7a″):

(7) a′. They *each* have four feet.

a″. They have four feet *in total*.[18]

[18]One who says (7a) as an answer to (8b) gives an incorrect answer, for (7a″) (as said about Chitu and Dilu) is false. But this does not mean that one cannot give (7a) as an answer (correct or incorrect) to the question.

Similarly, (7b) is ambiguous between 'They *each* have eight feet' and 'They have eight feet *in total*.' And we can take the ambiguity of (7a) and (7b) to arise from a systematic ambiguity of their predicates: 'have four feet' and 'have eight feet'. For example, 'have four feet' can mean either *each have four feet* or *have four feet in total*. As used to mean the former, it can be analyzed as, roughly, 'be such that *every one of* them has four feet';[19] as used to mean the latter, it can be analyzed as, roughly, 'be such that their feet (i.e., the things that are feet of any of them) are four'.

Say that a predicate is *distributive*, if it is true of some things (e.g., Chitu and Dilu) if and only if it is true of every one of them (e.g., both Chitu and Dilu). And say that a predicate used in a specific sense figures *distributively*, if the predicate as so used is true of some things if and only if it is true of every one of them. Then the predicate 'be equine' is distributive.[20] But 'be two horses' is not: the predicate is true of Chitu and Dilu (they are two horses), but not of Chitu (Chitu is not two horses but one horse). Now, consider two unequivocal predicates that the uses of 'have four feet' in (7a) amount to:

P1. each have four feet (viz., be such that every one of them has four feet)[21]

P2. have four feet in total

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[19]Or 'be such that *every one of* them is such that its feet (i.e., the things that are feet of it) are four'.

[20]Similarly, the Chinese predicate of (3a)–(3b) is distributive. So is its English counterpart, the predicate of which the singular form 'is a horse' while its plural form is 'are horses'. (The predicate is equivalent to 'be one or more horses'.)

[21]This predicate is an example of what I call *neutral expansions*. See, e.g., Yi (2005, 481–485).

P2 is not distributive. The predicate is true of Chitu and also of Dilu (Chitu, for example, has four feet), but not of the two (they have eight feet in total, not four). By contrast, P1 is distributive. For example, the predicate is true of Chitu and Dilu (*they* each have four feet) and is also true of every one of them (for Chitu has four feet and so does Dilu).[22] So we can say that the predicate 'have four feet' has the *distributive*/*non-distributive* ambiguity.[23]

We can now turn to the two-horse argument, which consists of (5a)–(5c). Consider (5b) and (5c). Like (7a), both statements have two readings matching the distributive and non-distributive uses of 'have four feet'. We can formulate the two readings of (5b) as follows:

(5) b′. Any things that are horses (*each*) *have four feet*.

b″. Any things that are horses *have four feet* (*in total*).

Similarly, we can formulate the two readings of (5c) as follows:

(5) c′. Any things that are two horses (*each*) *have four feet*.

c″. Any things that are two horses *have four feet* (*in total*).

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[22]P1 (i.e., 'each have four feet') is true of anything (i.e., any one thing) that the singular 'has four feet' is true of, and *vice versa*. We can explain this by using the logical truth 'Something *is one of* something if and only if the former is identical with the latter' (see Theorems 3[b] and 4[a] in Yi (2006, 264f)).

[23]Most non-distributive predicates are usually called *collective predicates*, for 'be two', for example, is true of many things (e.g., Chitu and Dilu) *collectively* or *taken together*. But this term is misleading, for most distributive predicates (e.g., 'be equine') are also true of many things taken together although they are *also* true of every one of the many things they are true of. For example, 'be equine' is true of Chitu and Dilu *collectively* and also true of them *individually* (i.e., true both of Chitu and of Dilu). Similarly, P1 is true of Chitu and Dilu collectively as well as individually.

Like the two readings of (7a), these readings of (5b) and (5c) are not equivalent. (5b′) and (5c′) are true (every one of any horses has four feet), but (5b″) and (5c″) are false (Chitu and Dilu, which are two horses, for example, have eight feet in total).

If so, which readings do (5b) and (5c) have in the two-horse argument? They must be read as (5b′) and (5c″), respectively, for (5b) is meant to be true while (5c) is meant to be false. If so, they cannot combine with (5a) to yield an instance of Plural Barbara, which requires the predicate replacing 'are-*R*' to figure without equivocation in the second premiss and conclusion.

The two-horse argument, we have seen, is not a proper instance of Plural Barbara because it involves equivocation of a key predicate: 'have four feet'. We can get a proper instance of the form by using the predicate in the same sense in (5b) and (5c), either distributively or non-distributively. This does not pose a threat to Plural Barbra, either, because the arguments we can get by doing so are equally valid. (5a) and (5b′) imply (5c′), and (5a) and (5b″) imply (5c″).[24]

This completes my account of the logical difference between the two arguments contrasted in the Horse Passage. On this account, the one-horse argument is valid because it is an instance of Plural Barbara (or its Chinese counterpart) but the two-horse argument is not because it involves equivocation. For the Chinese counterpart of 'have four feet' has the same ambiguity as the English phrase. One might use it distributively for two horses (e.g., Chitu and Dilu) because they *each* have four feet. Or one might use it non-distributively, in which case it is not true of two horses because

[24]In (4a) and (4b), both of which are meant to be true in [H2], 'have four feet' must be taken to figure interchangeably with 'each have four feet' to yield an instance of Plural Barbara. The statements (taken together) have three other readings, but those readings, too, yield valid arguments. One of the resulting three arguments is an instance of Plural Barbara (albeit one with a false premiss), and the other two (which are not instances of the form) are also valid because 'Any things that are *one horse* each have four feet' and 'Any things that are *one horse* have four feet in total' are logically equivalent.

they have eight feet (not four) *in total*. The two-horse argument, which is meant to be invalid, involves both uses: the distributive use in the second premiss and the non-distributive use in the conclusion.

In *Later Mohist Logic, Ethics and Science* (1978), a monument in the study of the dialectical chapters of the *Mozi*, A. C. Graham rightly notes that the Horse Passage draws attention to the distributive/non-distributive distinction. He says that the passage "concerns the distinction . . . between distributive and collective" (*ibid*., 493). But he does not relate the distinction to the ambiguity of 'have four feet'. Instead, he uses it to draw a distinction between 'be one' and 'be two'.[25] He would probably take the same distinction to hold between 'be one horse' and 'be two horses', and his point in the statement quoted above might be that the logical difference between the one-horse and two-horse arguments arises because the second (unlike the first) involves a non-distributive predicate: 'be two horses'. And those who give this account might conclude that the Horse Passage shows that Plural Barbara is not a valid form: while its instances involving only distributive predicates are valid, the form has invalid instances involving non-distributive predicates.

But this is not correct. The two-horse argument, we have seen, is not a proper instance of Plural Barbara. And all proper instances of the form, as noted above, are valid. Such instances include (6) and the two variants of the two-horse argument that we can get by eliminating the equivocation in (5b) and (5c):

(a) the argument that consists of (5a), (5b′), and (5c′);

[25]He says: the Mohists "distinguish what we should call the distributive and the collective use of words. Although we cannot say of two objects 'They are both two' . . . we can say that in some respect 'They are both one'" (*ibid*., 37).

(b) the argument that consists of (5a), (5b″), and (5c″).

Like the original two-horse argument, these arguments involve the non-distributive predicate 'be two horses',[26] and (6) involves *only* non-distributive predicates. Nevertheless, all these arguments (like the one-horse argument) are valid. One cannot explain this while rejecting the validity of Plural Barbara.

Moreover, the one-horse argument, too, involves a non-distributive predicate: 'be one horse'. Graham suggests that 'be one' is distributive by saying "we can say [of two objects] that in some respect 'They are both one'" (*ibid*., 37). This is correct. Although 'Chitu and Dilu are one' is false,[27] 'Chitu and Dilu are *both* one' is true—this statement is equivalent to 'Both Chitu and Dilu are one' and thus to 'Chitu is one and Dilu is one.' But this does not mean that 'be one' is a distributive predicate. It is not true of Chitu and Dilu (they are not one but two), but true of every one of them (Chitu is one and so is Dilu). So the predicate is not distributive. The same holds for 'be one horse'.

4. Semantics of Chinese nouns and predicates

Consider two English sentences:

(9) a. Chitu is a horse.

[26]And (b) involves another non-distributive predicate: 'have four feet (in total)'.

[27]The statement has a reading on which it is true, for 'be one' also has the distributive/non-distributive ambiguity: '(each) be one' and 'be one (as taken together)'. On the usual reading (the second), however, the predicate figures non-distributively, which is what I argue above.

b. Chitu and Dilu are horses.

In these sentences, the noun 'horse' and verb 'be' take singular and plural forms, and one cannot get (9b) from (9a) by simply replacing the singular term 'Chitu' with the plural term 'Chitu and Dilu'. But it is not the same in Chinese. We can get the Chinese counterpart of (9b) from that of (9a) by simply replacing the name *Chitu* with the phrase *Chitu yu Dilu* ('Chitu and Dilu'), for Chinese nouns and verbs themselves serve the roles of the singular or plural forms of their English counterparts.[28] The reason is that Chinese, we have noted, does not have a grammatical number system. For this reason, the noun *ma* ('horse') figures without taking a singular or plural form in (3a), as we have noted. This means that neither the singular (1a) or (1a′) nor the plural (4a) is closer to the Chinese statement in morphosyntactic structure than the other. As we have seen, however, the plural translation (4a) helps to explain the logic of the Horse Passage while the singular translations (1a) and (1a′) distort the logic. What is the reason for this difference?

The Chinese noun *ma* is a counterpart of the English noun 'horse', not its singular form, and figures where English would have either form of the noun. From the morphosyntactic point of view, then, the singular and plural forms of the English noun are equally close to (and removed from) the Chinese noun. But this does not mean that they have the same semantic standing as English counterparts of the Chinese noun. In semantic terms, the Chinese *ma* is closer to the plural form of the English noun than to its singular form. For the plural form, unlike the singular form, inherits the

[28]Thus the Chinese counterpart of 'is a horse' is the same as that of 'are horses'. The Chinese predicate is *ma*, for the noun *ma* ('horse') can figure as a predicate amounting to 'is a horse' and 'are horses' without addition of a copula, as in (3a) and (3b). (A copula (e.g., *nai*) might be added for special purposes, as in *nai ma* [be horse].) See note 8.

full semantic profile of the noun 'horse', of which *ma* is the Chinese counterpart. Let me explain.

To compare the semantics of the Chinese *ma* with those of the English noun 'horse' and its forms, it is necessary to distinguish the English noun from its singular form, which is its homonym. For this purpose, let me use 'HORSE' and 'horse*' for the noun and the singular form, respectively.

The singular form 'horse*' denotes *any one horse*. It denotes, e.g., Chitu. For 'Chitu is a horse' is true, and the sentence is true if and only if 'horse*' denotes something that the singular term 'Chitu' refers to (viz., Chitu). And the plural form 'horses' denotes *any horses*. It denotes, e.g., Chitu and Dilu (taken together), which are horses. For 'Chitu and Dilu are horses' is true, and the sentence is true if and only if 'horses' denotes some things (taken together) that the plural term 'Chitu and Dilu' refers to (viz., Chitu and Dilu).[29] How about the noun 'HORSE'? Its semantic profile must include those of the two forms, for their semantic profiles are, so to speak, projections of that of the noun to singular or plural contexts. So the noun denotes any one or more horses. It denotes any one horse as the singular form does, and any two or more horses as the plural form does.[30] It is the same with the Chinese noun *ma*. It figures in the Chinese counterparts of both (9a) and (9b) (without taking a singular or plural form). Thus *ma* denotes Chitu (as 'horse*' does), and also denotes Chitu and Dilu taken together (as 'horses' does). And it must denote any one or more horses because (3a) and (3b), for example, are true.

The Chinese *ma*, we have seen, have the same semantic profile as the English 'HORSE'.

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[29]Similarly, 'Chitu and Dilu are two horses' is true if and only if 'two horses' denotes Chitu and Dilu (taken together), and this holds if and only if both 'two' and 'horses' denote them (taken together). (Note that 'two' is not distributive.)

[30]The account of the semantics of 'horse' and its singular and plural forms sketched above is based on my treatment of plural constructions as devices for talking about the many. For the treatment, see, e.g., Yi (2005; 2006).

And it has the same semantic profile as the plural form 'horses', for this has the full semantic profile of 'HORSE'.

The *Oxford English Dictionary* defines 'plural' as "denoting more than one" (OED 2016). This suggests that 'horses', for example, cannot denote one horse (e.g., Chitu) although it can denote two or more horses taken together. But this is not correct. If it is, 'one or more horses' must be equivalent to 'two or more horses'[31] and (9b) to 'Chitu and Dilu are two or more horses.' But they are not. If John has only one horse, 'John has one or more horses' is true but 'John has two or more horses' is false. And (9b) is logically equivalent to 'Chitu is a horse and Dilu is horse', which does not imply 'Chitu and Dilu are two or more horses' because it is logically compatible with 'Chitu is identical with Dilu.' Moreover, 'Dilu and Yuyan are one (and the same) horse', like 'Cicero and Tully are one (and the same) person', is true ('Yuyan' is another name of Dilu), and yet one can correctly say 'Dilu and Yuyan are horses.' This means that 'horses' can denote Dilu, which is only one horse. And we can see that the plural form must denote any other horse as well. Like 'HORSE' and *ma*, then, the plural 'horses' denotes any one or more horses.

By contrast, the singular 'horse*' does not have the full semantic profile of the noun. Its use is limited to singular contexts (e.g., 'Chitu is a horse', 'Dilu is a horse', etc.), and its semantic profile can be taken to result from curtailing that of the noun for singular contexts. Although it is homonymous with the noun, we can take it to result from adding to the noun a silent morpheme, '-φ', as in 'HORSE-φ'. Unlike the plural morpheme '-s', which serves essentially as an agreement marker without semantic significance, the silent morpheme is semantically potent: it turns a noun

[31]For the left disjunct of 'one or more' (i.e., 'one') would be incompatible with the plural 'horses', if 'horses' could not denote any one thing (e.g., Chitu).

denoting any one or more of the things of a certain kind (e.g., the horses) to a form that cannot denote two or more of those things (e.g., two horses) taken together.

Now, it would be useful to compare the two translations of the Horse Passage: [H1] and [H2]. Although it is less colloquial than [H1], as we have seen, [H2] helps to clarify the logic of the Horse Passage. Moreover, we can see that it is a better translation. Consider (3a), which figures as the first premiss of the one-horse argument. The translations of (3a) in [H1] and [H2] amount to the singular (1a′) and the plural (4a), respectively. (4a) has the same logical content as (3a), but (1a′) falls short. Consider (10a)–(10b):

(10) a. If Chitu and Dilu are one horse, they are horses.

b. If Dilu and Yuyan are one horse, they are horses.

The Chinese counterparts of (10a)–(10b) are instances of (3a), which involves implicit universal quantification. Similarly, (10a)–(10b) are instances of the plural (4a). But neither is an instance of the singular (1a′). Similarly, the singular translations (1b) and (1c) of the second premiss and conclusion of the one-horse argument fail to capture their logic.[32]

4. Concluding Remarks

[32]Some might object that (4a) also falls short because its instances do not include the singular 'If Chitu is one horse, it is a horse.' On my account of plural constructions, however, the plural quantifier 'Any things' is equivalent to 'Any one or more things' and plural quantifications (e.g., (4a)) have singular instances as well as plural instances. See, e.g., Yi (2016, 267).

A key predicate figuring in the two arguments contrasted in the Horse Passage, as we have seen, has the distributive/non-distributive ambiguity. The predicate amounts to the English predicate 'have four feet'. This can be used as a short for '(each) have four feet' (P1) or for 'have four feet (in total)' (P2). The same holds for the Chinese predicate.

Consider two English arguments involving the predicate 'have four feet':

(11) Any horses have four feet.
Chitu is a horse.
:. Chitu has four feet.

(12) Any horses have four feet.
Chitu and Dilu are horses.
:. Chitu and Dilu have four feet.

Although (11) is a valid argument, (12) has a reading on which it is not. For the argument might be taken to involve the distributive use of 'have four feet' (or P1) in the first premiss and the non-distributive use (or P2) in the conclusion. If so, why does (11) not have the same ambiguity?

I think (11) is also ambiguous because it involves the same predicate. But this does not give rise to a *logical* difference for (11). The two readings of the predicate (P1 and P2) coincide with regard to any *one* thing (e.g., Chitu): P2 denotes anything (i.e., any one thing) that P1 denotes, and

vice versa.[33] Thus (11) is valid in all four readings resulting from disambiguating the predicate.[34] And one might ignore the distributive/non-distributive ambiguity as long as the predicate figures with the singular form: 'has four feet'. So some might explain the validity of (11) by reducing it to its singular cousin where the predicate figures only with the singular form:

(11′) Any horse *has four feet*.

Chitu is a horse.

:. Chitu *has four feet*.

On their explanation, (11) is valid because (11′) is valid and 'Any horses have four feet' is equivalent to 'Any horse has four feet.'

But this does not help to explain the validity or invalidity of (12) on the various readings. Moreover, the plural 'Any horses have four feet' is not equivalent to the singular 'Any horse has four feet.' Although the singular statement is true on both readings, the plural statement has a reading on which it is false: 'Any horses have four feet (in total)' is false because two horses (e.g., Chitu and Dilu) have eight feet in total, not four. On this reading, too, (11) is a valid argument, as we have noted. But one cannot explain this by reducing the argument to (11′).

To give a proper account of the logical difference between (11) and (12), then, it is necessary to develop an account of logic that one can apply to plural constructions of English as well as its

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[33]I.e., something is such that every one of it has four feet if and only if it has four feet in total.

[34]For example, 'Chitu is a horse' and 'Any horses *are such that every one of them has four feet*' imply 'Chitu *has four feet in total*.'

singular fragment. A major task in developing such an account is to give an adequate account of the logical difference between distributive and non-distributive predicates and the related ambiguity of predicates that can be true of two or more things, such as 'have four feet'. A passage of the *Mozi*, we have seen, has one of the earliest discussions of logical problems arising from the distributive/non-distributive distinction.[35] By giving a sharp formulation of such a problem, the passage highlights the need to attend to and control the distinction in studies of logic and semantics.

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[35]Plato discusses the non-distributivity of '(be) one' and 'be (two)' in the *Hippias Major* (301d–302b) (see, e.g., Woodruff's translation in Plato (1982, 27f)).

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Appendix 1: *Er* and *liang*

The first premiss and conclusion of the two-horse argument have different Chinese characters for two: *er* (the premiss) and *liang* (the conclusion). But *liang* is often used as an alternative form of *er*, which figures in the main series of Chinese numerals as the numeral for two.[36] So I treat *liang* as a mere variant of *er* and take the argument to be an apparent instance of Plural Barraba. But *liang* has a somewhat different use. Dobson says that it is also used as a numeral of "group form, 'two conceived as a pair'" in Archaic (or Old) Chinese (*ibid*., 497; cf. 13). And Graham takes it to figure in the argument as an "adjunct" that "marks off a pair from other things", and translates *liang ma* [two/pair horse] as "a pair of horses (taken together)" (1978, 192).

On this reading of *liang*, the two-horse argument can be formulated as follows:

(5*) Any things that are *two* horses are horses. (=(5a))

Any things that are horses have four feet. (=(5b))

:. Any things that are *a pair of* horses (*taken together*) have four feet.

This is not an instance of Plural Barbara and fails to draw even apparent syntactic parallels with the one-horse argument. But it results from contracting two (apparent) instances of Plural Barbara:

[36]See, e.g., Dobson (1974, 13 and 497).

(5′) Any things that are *a pair of* horses (*taken together*) are two horses.

Any things that are two horses are horses. (=(5a))

:. Any things that are *a pair of* horses (*taken together*) are horses.

(5″) Any things that are *a pair of* horses (*taken together*) are horses.

Any things that are horses have four feet. (=(5b))

:. Any things that are *a pair of* horses (*taken together*) have four feet.

On this analysis, (5*) rests on an implicit assumption: the first premiss of (5′).

The Horse Passage, on the analysis, holds that the conclusion of (5*) is false while its two explicit premisses and the implicit assumption are all true. This poses a problem for the validity of Plural Barbara. But we can resolve this problem as well by applying the account of invalidity of the argument consisting of (5a)–(5c) presented in section 3. Like this argument, (5*) involves equivocation of 'have four feet': it is used distributively in the second premiss but non-distributively in the conclusion. This equivocation in (5*) results from the same equivocation in (5″). This means that (5″) is not a proper instance of Plural Barbara.

Appendix 2: The mass noun thesis

In analyzing the Horse Passage, I treat the Chinese noun *ma* ('horse') as a count noun that has the same semantic profile as its English counterpart: the noun 'horse' (or 'HORSE') (see §4). Like the English noun, it denotes any one or more of some individuals belonging to the same kind (viz., the

horses). And in Classical Chinese, the written language of the classical literature that includes the *Mozi*, the noun can directly combine with numerals, as in (3a)–(3b), where it combines directly with *yi* ('one') and *er* ('two'). This is sufficient (if not necessary) for classifying it as a count noun. Although the noun does not take a singular or plural form, this is not because it is a mass noun. It does not take such a form for the same reason that the Chinese verb for love, for example, does not take a singular or plural form in the Chinese counterparts of 'He *loves* her' and 'They *love* her.' The reason is that Chinese, unlike English, does not have a grammatical number system. In languages without a grammatical number system, count nouns have neither singular nor plural forms.

Now, many linguists and philosophers (e.g., Hansen (1983; 1992)) deny that Chinese has any count nouns. They hold the *mass noun thesis* for Chinese, the thesis that all Chinese common nouns are mass nouns, which cannot directly combine with numerals. For in modern dialects of Chinese (e.g., Mandarin), common nouns cannot directly combine with numerals: the noun *ma*, for example, requires special expressions called *numeral classifiers* (CL) to combine with numerals, as in *yi pi ma* ('one horse' [one CL horse]). They argue that the mass noun thesis yields a good explanation of this requirement (the classifier requirement) and some other features of Chinese nouns (e.g., the lack of plural morphology), which they take to be related to the classifier requirement. But there is an obvious problem for holding this thesis for Classical Chinese: Classical Chinese does not have a substantial numeral classifier system and has nouns that can combine directly with numerals, as *ma* ('horse') does in (3a)–(3b). Moreover, I think the thesis has problems even with Modern Chinese. In modern dialects of Chinese, as in Classical Chinese, some nouns (e.g., *ma*) can combine directly with Chinese counterparts of some counting expressions that relate to the number: 'countless', 'a large number of', 'a majority of', etc. Such nouns cannot be mass nouns, and they

must be taken to refer to one or more individuals that belong to the same kind (e.g., horses). Thus I think both Classical and Modern Chinese have count nouns. And I propose a syntactic criterion of Chinese count nouns: Chinese count nouns, unlike mass nouns, can combine directly with the Chinese counterparts of 'countless', 'a large number of', and 'a majority of'. This criterion is applicable to both Classical and Modern Chinese and rules that *ma* is a count noun in both stages of the Chinese language.[37]

Appendix 3: The sequel to the Horse Passage

The translations of the Horse Passage given in the main body of this article omits the sequel to the passage that contrasts two other arguments that concern one horse and two horses. In line with the plural translation of the horse passage, [H2], we can translate the sequel as follows:

> [H*] . . . . Any things that are *one horse* are horses [just as any things that are *two horses* are horses]. If [the] horses are such that some of them are white, it means that there are some things that are *two horses* and some of them are white, but not that there are some things that are *one horse* and some of them are white. One [of each of the pairs of arguments discussed above] is correct but one [i.e., the other] is not. (My italics)

[37]For more about my accounts of numeral classifiers and the mass/count distinction, see Yi (2009; 2011a; 2014; forthcoming 1; forthcoming 2), which have references to works that propose the mass noun thesis. Incidentally, the Chinese counterpart of (1b) and (4b), *ma shi zhu* ('Horses (each) have four feet' [horse four feet]), raises further problem for the mass noun thesis, as Fung (2007, 527f) and Fraser (2007, 433) point out. See also Harbsmeier (1991).

Like the Horse Passage, this passage contrasts two arguments one of which results from replacing 'two horses' in the other with 'one horse':

(13) Any things that are *one horse* are horses.

The horses are such that some of them are white.

:. There are some things that are *one horse* and some of them are white.

(14) Any things that are *two horses* are horses.

The horses are such that some of them are white.

:. There are some things that are *two horses* and some of them are white.

If we take the 'some' in 'some of them are white' to be short for 'some *but not all*', we can see that (14) is a valid argument. But (13) is not.[38] Although both of its premisses are true, its conclusion is false—no one horse can include both a white horse and a non-white horse, while two horses can include both a white horse and a non-white horse (e.g., a black horse). What is the reason for this difference between (13) and (14)?

Note that the invalidity of (13), unlike that of the two-horse argument in the Horse Passage, is not due to equivocation. This means that the validity of (14) does not stem from the form of argument that it shares with (13):

[38]Suppose that some of the horses are white while some of them are not. Then one of the former and one of the latter are two horses, and some but not all of them are white.

(14*) Any things that are-P are horses.

The horses are such that some [but not all] of them are white.

:. There are some things that are-P and some [but not all] of them are white.

This is not a valid form of argument. (13) is a counterexample to its validity. And we can confirm the invalidity of (14*) by considering some other arguments, such as (15a)–(15c):

(15a) Any things that are *black horses* are horses.

The horses are such that some [but not all] of them are white.

:. There are some things that are *black horses* and some [but not all] of them are white.

(15b) Any things that are *two black horses* are horses.

The horses are such that some [but not all] of them are white.

:. There are some things that are *two black horses* and some [but not all] of them are white.

(15c) Any things that are *three horses* are horses.

The horses are such that some [but not all] of them are white.

:. There are some things that are *three horses* and some [but not all] of them are white.

These argument results from replacing 'are-P' in (14*) with 'are *black horses*', 'are *two black horses*', and 'are *three horses*' respectively. But they are all invalid. Although both premisses of

(15a) are true, its conclusion cannot be true—no black horses can include any white horse. It is the same with (15b). (15c) is somewhat different: its conclusion happens to be true. But this does not mean that the argument is valid. If there are only two horses and one of them is white and the other black, then, all its premisses are true but the conclusion are false. Like the others, then, (15c) is also invalid. So the validity of (14) rests on special logical features of the predicate 'are two horses' that most other predicates do not share:

(16) Something that is a horse is not identical with something that is also a horse if and only if the former and the latter *are two horses*.[39]

[39]We can use plural logic to show that this is a logical truth. See, e.g., Yi (1998; 1999, 188f).