On the very idea of imperative inference

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Outline

Rescher (1966) on an imperative inference (1/3)

Always say 'please' to John when you ask him for the bread!!
Ask John for the bread now!
———
Say 'please' to John now!

Rescher, N. (1966), *The Logic of Commands*, p. 77.

Are the premises and the coclusion just imperative sentences?

Or, do they stand for acts of commanding?

Or, do they stand for what are commanded?

Rescher (1966) on an imperative inference (2/3)

Always say 'please' to John when you ask him for the bread!!
Ask John for the bread now!
———
Say 'please' to John now!

The inference may be characterized as 'valid' in the sense that its conclusion is tacitly or implicitly contained in its premises so that (*inter alia*):

Rescher (1966) on 'command inference' (3/3)

(i) *Anyone* who overtly *gives* the premiss commands may legitimately claim (or be claimed) to have implicitly given the command conclusion.

(ii) *Anyone* who overtly *receives* the premiss commands may legitimately claim (or be claimed) to have implicitly received the command conclusion.

(iii) Any course of action on the part of their common recipient which terminates the premiss commands cannot fail to terminate the command conclusion.

Rescher, N. (1966), *The Logic of Commands*, pp. 77-78.

Conflicting commands (1/2)

Suppose you are on a team of researchers and the leader of the group commands you to give a presentation of the results of the research project the team has been engaged in at a one day international workshop to be held in São Paulo on August 9 next year.

Suppose, in addition, you are also a member of a political group and you have received a letter from the guru of the group in which she commands you to join an important demonstration in Sapporo on the very same day.

Although the time in São Paulo is 12 hours behind the time in Sapporo, you will not be able to attend the workshop in São Paulo if you join the demonstration in Sapporo.

Conflicting commands (2/2)

The difficulty here can be said to be just a contingent difficulty.

But what if your guru should command you not to go to São Paulo on August 9 next year?

Then you will have genuine logical incompatibility.

The language of MDL$^+$III (Yamada, 2008a)

Definition of $\mathcal{L}_{\mathrm{MDL^+III}}$

Take a countably infinite set Aprop of proposition letters, and a finite set I of agents, with p ranging over Aprop, and i, j, k over I. The language $\mathcal{L}_{\mathrm{MDL^+III}}$ of the Multi-agent Deontic Logic MDL$^+$III is given by:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \Box\varphi \mid O_{(i,j,k)}\varphi$$

Obligations in MDL$^+$III (Yamada, 2008a)

The language of MDL$^+$III has formulas of the following form:

$O_{(i,j,k)}\varphi$.

It means that it is obligatory upon an agent i with respect to an agent j by the name of an agent k to see to it that φ,

where

- i is an agent who owes the obligation (obligor)
- j is an agent to whom the obligation is owed (obligee)
- k is an agent who create the obligation.

Models for MDL$^+$III

definition

By an $\mathcal{L}_{\text{MDL}^+\text{III}}$-model, we mean a tuple $M = \langle W^M, A^M, \{D^M_{(i,j,k)} \mid i,j,k \in I\}, V^M\rangle$ where:

1. W^M is a non-empty set (heuristically, of 'possible worlds' or 'states')
2. A^M is a binary relation such that $A^M \subseteq W^M \times W^M$
3. $D^M_{(i,j,k)} \subseteq A^M$
4. V^M is a function that assigns a subset $V^M(p)$ of W^M to each proposition letter $p \in \mathrm{Aprop}$.

Obligations in MDL$^+$III (2)

Given an $\mathcal{L}_{\text{MDL}^+\text{III}}$-model, truth definition for the formulas of $\mathcal{L}_{\text{MDL}^+\text{III}}$ is given in a completely standard way by associating $\Box$ with A^M and $O_{(i,j,k)}$ with $D^M_{(i,j,k)}$.

The clause for deontic formulas reads as follows:

$M, w \models_{\text{MDL}^+\text{III}} O_{(i,j,k)}\varphi$
iff for any v such that $\langle w, v\rangle \in D^M_{(i,j,k)}$, $M, v \models_{\text{MDL}^+\text{III}} \varphi$.

Two comments on MDL$^+$III

Since we need to be able to deal with conflicting commands, we do not accept D Axiom in MDL$^+$III.

MDL$^+$III inherits various problematic features of standard deontic logic. Our use of MDL$^+$III does not reflect our theoretical commitment to these features. We are only trying to keep things as simple as possible at this early stage of the development of dynamified deontic logic. We are thinking of dynamifying other systems of deontic logic as our future tasks.

Dynamifying MDL$^+$III (Yamada, 2008a)

Definition of $\mathcal{L}_{\text{DMDL}^+\text{III}}$

Take the same countably infinite set Aprop of proposition letters, and the same finite set I of agents as before, with p ranging over Aprop, and i, j, k over I. The language $\mathcal{L}_{\text{DMDL}^+\text{III}}$ of the Dynamified Multi-agent Deontic Logic DMDL$^+$III is given by:

$$\begin{array}{lcl} \varphi & ::= & \top \mid p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \Box\varphi \mid O_{(i,j,k)}\varphi \mid [\pi]\varphi \\ \pi & ::= & \mathrm{Com}_{(i,j)}\psi \mid \mathrm{Prom}_{(i,j)}\psi \end{array}$$

Acts of Commanding and Promising in DMDL$^+$III

$[\mathrm{Com}_{(i,j)}\varphi]\psi$

After an agent i gives an addressee j a command to the effect that j should see to it that φ, ψ holds.

$[\mathrm{Prom}_{(i,j)}\varphi]\psi$

After an agent i gives an addressee j a promise to the effect that i will see to it that φ, ψ holds.

Interpreting dynamic formulas

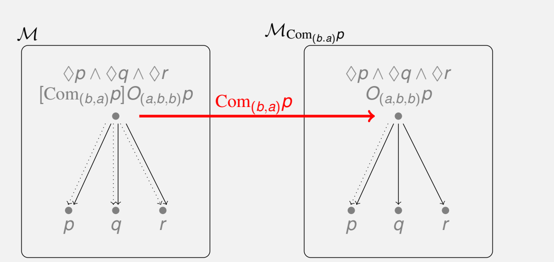
In the truth definition for the language of DMDL$^+$III, the added dynamic formulas are interpreted by the following clauses:

$$M, w \models_{\text{DMDL}^+\text{III}} [Com_{(i,j)}\varphi]\xi \;\text{ iff }\; M_{Com_{(i,j)}\varphi}, w \models_{\text{DMDL}^+\text{III}} \xi$$
$$M, w \models_{\text{DMDL}^+\text{III}} [Prom_{(i,j)}\varphi]\xi \;\text{ iff }\; M_{Prom_{(i,j)}\varphi}, w \models_{\text{DMDL}^+\text{III}} \xi,$$

where

1. $M_{Com_{(i,j)}\varphi}$ is the $\mathcal{L}_{\text{MDL}^+\text{III}}$-model obtained from M by replacing $D^M_{(j,i,i)}$ with its subset $\{\langle x, y\rangle \in D^M_{(j,i,i)} \mid M, y \models_{\text{DMDL}^+\text{III}} \varphi\}$, and
2. $M_{Prom_{(i,j)}\varphi}$ is the $\mathcal{L}_{\text{MDL}^+\text{III}}$-model obtained from M by replacing $D^M_{(i,j,i)}$ with its subset $\{\langle x, y\rangle \in D^M_{(i,j,i)} \mid M, y \models_{\text{DMDL}^+\text{III}} \varphi\}$.

Your boss's act of commanding in DMDL$^+$III



$\mathcal{M}, w \models [\mathrm{Com}_{(b,a)}p]O_{(a,b,b)}p$ iff $\mathcal{M}_{\mathrm{com}_{(b,a)}p}, w \models O_{(a,b,b)}p$.

How acts of commanding and promising work

The CUGO Principle:

If φ is a formula of MDL$^+$III and is free of occurrences of the modal operator $O_{(j,i,i)}$, $[\mathrm{Com}_{(i,j)}\varphi]O_{(j,i,i)}\varphi$ is valid.

The PUGO Principle:

If φ is a formula of MDL$^+$III and is free of occurrences of the modal operator $O_{(j,i,i)}$, $[\mathrm{Prom}_{(i,j)}\varphi]O_{(i,j,i)}\varphi$ is valid.

Conflicting commands

By CUGO Principle, we have:

A contingent dilemma

$[\mathrm{Com}_{(b,a)}p][\mathrm{Com}_{(g,a)}q](O_{(a,b,b)}p \wedge O_{(a,g,g)}q)$,

where

p : a gives a talk in a workshop in São Paulo on August 9 2012.
q : a joins a demonstration in Sapporo on August 9 2012.

Moreover, we have:

$[\mathrm{Com}_{(b,a)}p][\mathrm{Com}_{(g,a)}\neg p](O_{(a,b,b)}p \wedge O_{(a,g,g)}\neg p)$.

Rescher (1966) on an imperative inference (1/2)

Always say ‘please’ to John when you ask him for the bread!!
Ask John for the bread now!
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Say ‘please’ to John now!

The inference may be characterized as ‘valid’ in the sense that its conclusion is tacitly or implicitly contained in its premises so that (*inter alia*):

Rescher (1966) on ‘command inference’ (2/2)

(i) *Anyone* who overtly *gives* the premiss commands may legitimately claim (or be claimed) to have implicitly given the command conclusion.

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Rescher, N. (1966), *The Logic of Commands*, pp. 77-78.

Why call it a “command conclusion”?

See to it that $(p \wedge q)$!
———
See to it that p !

What does the “command conclusion” tell us?

It tells us what the commandee has to do in order to obey (or terminate) the command(s) actually given.

But do we need “command conclusion” here?

Deontic formulas can be used to say what the commandee has to do in order to obey (or terminate) the command(s) given to her. Note that by CUGO Principle, we have:

(1) $[\mathrm{Com}_{(m,c)}(p \wedge q)]O_{(c,m,m)}(p \wedge q)$
(2) $[\mathrm{Com}_{(m,c)}p]O_{(c,m,m)}p$

Note also that we can derive (3) from (1).

(3) $[\mathrm{Com}_{(m,c)}(p \wedge q)]O_{(c,m,m)}p$

A way of reasoning about imperatives

(3) can be considered as telling us what Rescher’s “command conclusion” is supposed to tell us.

(3) $[\mathrm{Com}_{(m,c)}(p \wedge q)]O_{(c,m,m)}p$

See to it that $(p \wedge q)$!
———
See to it that p !

A way of reasoning about imperatives

Since (3) is valid, for any $\mathcal{L}_{\mathrm{MDL^+III}}$-model M and a world w of M, we have:

$M, w \models_{\mathrm{MDL^+III}} [\mathrm{Com}_{(m,c)}(p \wedge q)]O_{(c,m,m)}p$

This implies:

$M_{\mathrm{Com}_{(m,c)}(p \wedge q)}, w \models_{\mathrm{MDL^+III}} O_{(c,m,m)}p$

This seems to tell us what our “imperative inference” is meant to tell us.

See to it that $(p \wedge q)$!
———
See to it that p !

A way of reasoning about imperatives

Note that the expressions of the form $\mathrm{Com}_{(i,j)}\varphi$ are terms that stand for types of acts of commanding. As a result, they can be neither premises nor conclusions of inferences by themselves.

Dynamified deontic logic shows that we can reason about acts of commanding nonetheless.

Dynamified deontic logic may be a way of doing what "imperative inference" is supposed to do.

The Involves Relation

We may define something like the "involves relation" much discussed in Situation Theory:

$\pi_1 \Rightarrow \pi_2$
iff $\models [\pi_2]\varphi$ implies $\models [\pi_1]\varphi$.

Then, as an instance, we can prove:

$\mathrm{Com}_{(i,j)}(p \wedge q) \Rightarrow \mathrm{Com}_{(i,j)}p$.

Conflicting speech acts

By CUGO Principle and PUGO Principle, we have:

A contingent dilemma

$[\mathrm{Prom}_{(a,s)}p][\mathrm{Com}_{(g,a)}q](O_{(a,s,a)}p \wedge O_{(a,g,g)}q)$,

where

p : a gives a talk in a workshop in São Paulo on August 9 2012.
q : a joins a demonstration in Sapporo on August 9 2012.

Or, again:

A dilemma

$[\mathrm{Prom}_{(a,s)}p][\mathrm{Com}_{(g,a)}\neg p](O_{(a,s,a)}p \wedge O_{(a,g,g)}\neg p)$.

On other speech acts

Differentiating illocutionary acts of commanding from perlocutionary acts that affects preferences

Dynamified deontic preference logic (Yamada 2008b).

Asserting, conceding, and their withdrawals

Dynamic logics of propositional commitments (Yamada, 2011).

Differentiating acts of requesting from acts of commanding

A dynamified deontic epistemic logic (Yamada, 2012).

The Involves Relation Again

In the dynamic logic of propositional commitments, we can derive another instance of the involves relation:

$\mathrm{Assert}_i(p \wedge q) \Rightarrow \mathrm{Assert}_i p$.

$\mathrm{Com}_{(i,j)}(p \wedge q)$ involves $\mathrm{Com}_{(i,j)}p$, while $O_{(j,i,i)}(p \wedge q)$ implies $O_{(j,i,i)}p$.

$\mathrm{Assert}_i(p \wedge q)$ involves $\mathrm{Assert}_i p$, while $[\text{a-cmt}]_i(p \wedge q)$ implies $[\text{a-cmt}]_i p$.

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